2014

NAME: KEY

Test: Gravity

Some equations you may need:

$$F = G\frac{m_1 m_2}{r^2} \qquad \frac{T^2}{R^3} = \frac{4\pi^2}{GM} \qquad v_e = \sqrt{\frac{2GM}{r}} \qquad U = -\frac{Gm_1 m_2}{r} \qquad E = -\frac{GmM}{2R}$$

$$F_c = \frac{mv^2}{r} \qquad K = \frac{1}{2}mv^2 \qquad L = I\omega \qquad \vec{\tau} = \vec{r} \times \vec{F} \qquad I = \sum mr^2 = \int r^2\,dm \qquad e = \frac{c}{R} \qquad T = \frac{2\pi}{\omega}$$

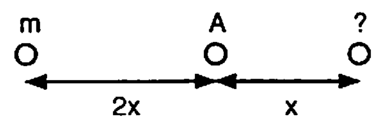
Some constants you may need:

$G = 6.67 \times 10^{-11}$ Nm²/kg² $\quad M_{earth} = 6 \times 10^{24}$ kg $\quad R_{earth} = 6.4 \times 10^{6}$ m $\quad D_{earth\text{-}sun} = 1.5 \times 10^{11}$ m

$M_{sun} = 2 \times 10^{30}$ kg $\quad R_{sun} = 7 \times 10^{8}$ m $\quad M_{moon} = 7.35 \times 10^{22}$ kg $\quad R_{moon} = 1.8 \times 10^{6}$ m

Multiple Choice: *Choose the letter of the best answer. 3 points each.*

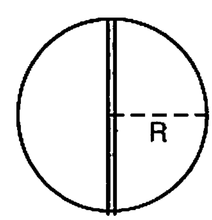
1. B Where could two objects have 0 gravitational potential energy?
   a. When they are touching each other.
   b. When they are infinitely far away.
   c. It's impossible because they would have the same center.
   d. As long as they are orbiting each other they would have 0 potential energy.



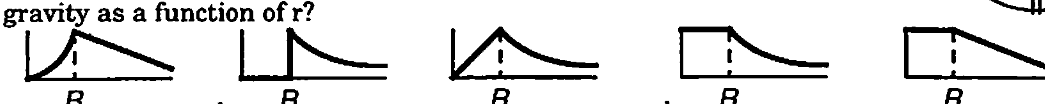
2. E A mass "m" is a distance of 2x to the left of A (shown above.) There is a second mass a distance x to the right of A. What should be the second mass so that the net force at A is 0?
   a. 2m. b. 4m. c. m. d. (1/2) m. e. (1/4) m.

3. A A mass M (with a radius R) is a certain distance away from a second mass m, as shown above. The gravitational attraction between the two masses is 80 N. If the size of M is doubled to 2R, but its density remains the same, what would be the force between the two masses?
   a. 640 N. b. 320 N. c. 160 N. d. 80 N. e. None of those.

Problems 4 and 5 refer to the following:
A planet of uniform density and radius R has a small tunnel somehow drilled through its center from pole to pole, as shown in the diagram to the right.



4. C Which of the following would be the best graph of the acceleration due to gravity as a function of r?



a. b. c. d. e.

5. B What would happen to a small rock at the North Pole if it were dropped into the tunnel?
   a. It fall back and forth at constant speed between the poles.
   b. It would undergo simple harmonic motion between the poles.
   c. It would hover over the hole weightless.
   d. It would fall to the center of the planet, and then get ripped apart by the strong tidal forces.

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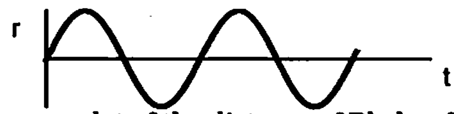
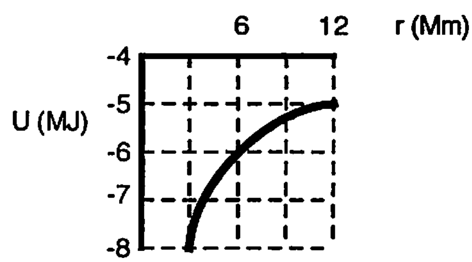
6. E Two identical objects (each mass m) are separated by a distance of r. How much work would it take to quadruple the distance between them?
a. $\frac{Gm^2}{4r}$ b. $\frac{Gm^2}{2r}$ c. $\frac{2Gm^2}{3r}$ d. $\frac{4Gm^2}{3r}$ e. $\frac{3Gm^2}{4r}$

7. B If we were to somehow lose the moon, what would happen to the tides here on earth?
a. There wouldn't be tides anymore. So sad.
b. There would still be tides, but they would be much smaller and more regular.
c. The tides would be become much greater.
d. Huh? There would be no effect whatsoever on the tides.



8. A The diagram above shows a plot of the distance of Phobos from Mars as a function of time after making a long series of telescopic observations. The best fit equation for the data is r = 9400 sin(19.7 t) where r is measured in km and t in days. What is the mass of Mars?
a. 6.5×10^{23} kg. b. 1.6×10^{22} kg. c. 1.3×10^{21} kg. d. 2.2×10^{27} kg. e. 2.2×10^{18} kg.



9. D The potential energy for an object near a planet of radius 3 Mm is given by the graph shown above. The object is dropped from a "height" of 3 Mm above the surface of the planet. What is its kinetic energy just as it hits the planet?
a. 8 MJ. b. 6 MJ. c. 4 MJ. d. 2 MJ. e. None of these.

10. E Imagine you are inside a uniform shell of mass M and radius R. If the mass of the shell were increased to 2M, what would happen to the force of gravity exerted by the shell on you?
a. It would increase to twice what it was.
b. It would increase to $\sqrt{2}$ what it was.
c. It would decrease to half what it was.
d. It would increase to 4 times what it was.
e. It wouldn't change at all.

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Problem Solving: *Show all work.* 12 @

orbit energy -6

$F_c = F_g$ -10

V_e -10

11. Imagine you throw a rock straight up with a speed of 8500 m/s from the surface of the earth. How high will it go? You can either give an answer to the center of the earth or the height above the surface of the earth, but make sure you tell me what you found. Feel free to ignore air resistance.

$$\frac{1}{2}mv^2 - G\frac{mM}{R} = -G\frac{mM}{r}$$

$$\frac{1}{2}v^2 - \frac{GM}{R} = -\frac{GM}{r}$$

$$\frac{v^2}{2GM} - \frac{1}{R} = \frac{1}{r}$$

$$\frac{(8500)^2}{2\cdot G\cdot(6\times10^{24})} - \frac{1}{6.4\times10^6} = -\frac{1}{r}$$

$$9.03\times10^{-8} - \frac{1}{6.4\times10^6} = -\frac{1}{r}$$

$$-\frac{1}{r} = -6.6\times10^{-8}$$

$$\boxed{r = 15{,}200{,}000 \text{ m (to center)} = 15{,}200 \text{ km}}$$

$= 8800$ km above

aphelion -3

used same distances -4

units in 3rd law -3

$F_c = F_g$ -9

only R -9

12. What is the fastest speed an asteroid could have in its orbit around the sun if it had an eccentricity of 0.25 and it took 7 years to go around the sun?

$$\frac{T^2}{R^3} = 1 = \frac{(7)^2}{R^3} \quad R^3 = 49 \quad R = 3.66 \text{ AU} \Rightarrow = 5.49\times10^{11} \text{ m}$$

$$\rho = R - c = R - eR$$

$$\rho = 3.66 - (.25)(3.66)$$

$$\rho = 2.74 \text{ AU} = 4.12\times10^{11} \text{ m}$$

$$E = K + U$$

$$-G\frac{\cancel{m}M}{2R} = \frac{1}{2}\cancel{m}v^2 - G\frac{\cancel{m}M}{\rho}$$

$$-\frac{GM}{R} = v^2 - \frac{2GM}{\rho}$$

$$v^2 = \frac{2GM}{\rho} - \frac{GM}{R}$$

$$v^2 = 2GM\left(\frac{1}{\rho} - \frac{1}{2R}\right)$$

$$v^2 = 2\cdot G\cdot M\cdot\left(\frac{1}{4.12\times10^{11}} - \frac{1}{2(5.49\times10^{11})}\right)$$

$$v^2 = 405{,}000{,}000$$

$$\boxed{v = 20{,}100 \text{ m/s}}$$

generic Algebra -2

wrong volume -2

13. Imagine a star rotates with a period of T. What is the minimum density of the star so that it just barely stays together?

$$\frac{T^2}{R^3} = \frac{4\pi^2}{GM}$$

$$\rho = \frac{M}{\frac{4}{3}\pi R^3}$$

$$\frac{\cancel{m}v^2}{\cancel{R}} = \frac{G\cancel{m}M}{R^{\cancel{2}}}$$

$$v^2 = \frac{GM}{R}$$

$$M = \frac{Rv^2}{G} = \frac{R}{G}\frac{4\pi^2R^2}{T^2}$$

$$\frac{M}{R^3} = \frac{4\pi^{\cancel{2}}}{GT^2} = \frac{4}{3}\cancel{\pi}\rho$$

$$\boxed{\rho = \frac{3\pi}{GT^2}}$$

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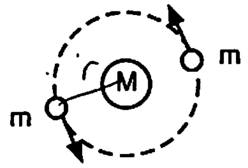
14. Two stars of mass m are both orbiting a larger star of mass M. Both the smaller stars are a distance r from the larger star, and are exactly on opposite sides of M. What is the period of oscillation of the little stars?



same orbit
∴ same period.

$$v = \frac{2\pi r}{T}$$

$$\Sigma F = \frac{mv^2}{r}$$

$$G\frac{mM}{r^2} + G\frac{m^2}{(2r)^2} = \frac{mv^2}{r}$$

$$\frac{GM}{r^2} + \frac{Gm}{4r^2} = \frac{v^2}{r} = \frac{4\pi^2 r^2}{T^2 r}$$

$$v = \frac{2\pi r}{\sqrt{\frac{4GM + Gm}{4r}}}$$

$$GM + \frac{1}{4}Gm = \frac{4\pi^2 r^3}{T^2} \Rightarrow T = \sqrt{\frac{4\pi^2 r^3}{G(M + \frac{1}{4}m)}}$$

Just K3L -10

$F_c = F_g$ but no 2nd mass -5

15. Derive Kepler's Second Law. Don't forget to impress me with the clarity of your presentation - which means explain what you are doing.

No central Force -3

did K3L -10